11.4

Are You Afraid of Ghosts?

Factored Form of a Quadratic Function

LEARNING GOALS

In this lesson, you will:

- Factor the greatest common factor from an expression.
- Write a quadratic function in factored form.
- Determine the x-intercepts from a quadratic function written in factored form.
- Determine an equation of a quadratic function given its x-intercepts.

KEY TERMS

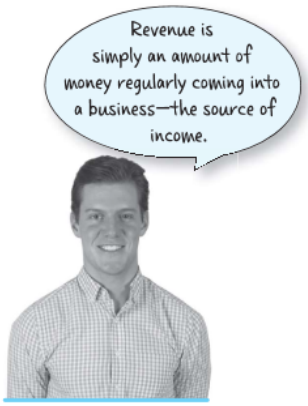
- factor an expression
- factored form

Touring allegedly "haunted" houses in the United States is a big business. Go to Tombstone, Arizona, where you're told that you can hear ghosts laughing and yelling. Or go to Hotel Jerome in Aspen, Colorado. This hotel is said to be haunted by a young boy. How about Bannack, Montana—a ghost town where you might meet a ghost named Dorothy and hear babies crying?

PROBLEM 1 Making the Most of the Ghosts

The Jacobson brothers own and operate their own ghost tour business. They take tour groups around town on a bus to visit the most notorious "haunted" spots throughout the city. They charge $50 per tour. Each summer, they book 100 tours at that price. The brothers are considering a decrease in the price per tour because they think it will help them book more tours. They estimate that they will gain 10 tours for every $1 decrease in the price per tour.

Let's consider the revenue for the ghost tour business. In this situation, the revenue is the number of tours multiplied by the price per tour.



1. Let x represent the change in the price per tour. Write an expression to represent the number of tours booked if the decrease in price is x dollars per tour. Write the expression so that the variable term comes first.

2. Write an expression to represent the price per tour if the brothers decrease the price x dollars per tour. Write the expression so that the variable term comes first.

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To **factor an expression** means to use the Distributive Property in reverse to rewrite the expression as a product of factors.

When factoring algebraic expressions, you can factor out the greatest common factor from all the terms.

Consider the expression $12x + 42$.

The greatest common factor of $12x$ and 42 is 6. Therefore, you can use the Distributive Property in reverse to rewrite the expression.

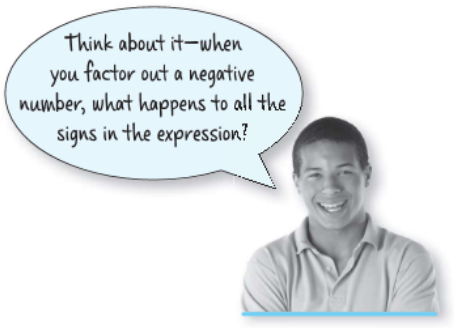
$$12x + 42 = 6(2x) + 6(7)$$
$$= 6(2x + 7)$$

So, the factored expression is $6(2x + 7)$.

3. Rewrite the expression from Question 1 by factoring out the greatest common factor.

When factoring an expression that contains a negative leading coefficient, it is a convention to factor out the negative sign. Remember that a coefficient is a number that is multiplied by a variable in an algebraic expression.

4. Rewrite the expression from Question 2 by factoring out −1.



You can write a quadratic function in factored form to represent the Jacobson brothers' revenue, $r(x)$, in terms of the decrease in price per tour, x.

A quadratic function written in **factored form** is in the form $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$.

5. Use your expressions from Questions 3 and 4 to first represent the revenue, $r(x)$, as the number of tours times the price per tour. Then write $r(x)$ in factored form.

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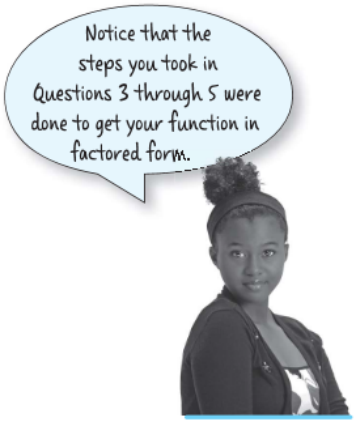
Revenue	=	Number of Tours	•	Price per Tour
↓		↓		↓
$r(x)$	=	_____	•	_____
$r(x)$	=	_____		



6. Graph $r(x)$ on a graphing calculator using the bounds $[-10, 50] \times [0, 10{,}000]$. Sketch and label your graph.

7. Use a graphing calculator to determine each key characteristic. Then, interpret the meaning of each in terms of this problem situation.

   a. x-intercepts

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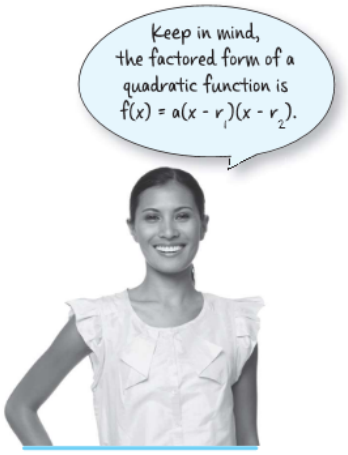
   b. y-intercept

   c. absolute minimum or absolute maximum



8. Compare your answers in Question 7 to the factored form of the function you wrote in Question 5. Which, if any, key characteristics do you think you can determine directly from the equation of the function when written in factored form?

PROBLEM 2 Exploring Factored Form

1. A group of students are working together on the problem shown.

Write a quadratic function in factored form to represent a parabola that opens downward and has zeros at (4, 0) and (−1, 0).

Maureen
My function is
$k(x) = -(x - 4)(x + 1)$.

Micheal
My function is
$d(x) = \frac{1}{2}(x - 4)(x + 1)$.

Tim
My function is
$m(x) = 2(x - 4)(x + 1)$.

Tom
My function is
$k(x) = -2(x - 4)(x + 1)$.

Dianne
My function is
$t(x) = -0.5(x - 4)(x + 1)$.

Judy
My function is
$f(x) = -(x + 4)(x - 1)$.

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a. Use your graphing calculator to graph each student's function. What are the similarities among all the graphs? What are the differences among the graphs.

b. How is it possible to have more than one correct function?

c. What would you tell Micheal, Tim, and Judy to correct their functions.

d. How many possible functions can you write to represent the given characteristics? Explain your reasoning.

2. For a quadratic function written in factored form $f(x) = a(x - r_1)(x - r_2)$:

a. what does the sign of the variable a tell you about the graph?

b. what do the variables r_1 and r_2 tell you about the graph?

3. Use the given information to write a quadratic function in factored form, $f(x) = a(x - r_1)(x - r_2)$.

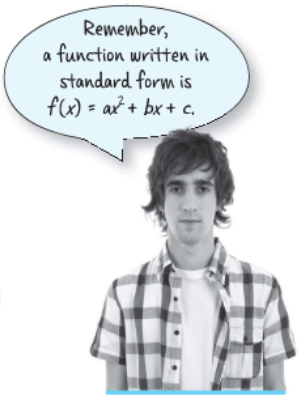
a. The parabola opens upward and the zeros are (2, 0) and (4, 0).

b. The parabola opens downward and the zeros are (−3, 0) and (1, 0).

c. The parabola opens downward and the zeros are (0, 0) and (5, 0).

d. The parabola opens upward and the zeros are (−2.5, 0) and (4.3, 0).

4. Compare your quadratic functions with your classmates' functions. How does the *a* value affect the shape of the graph?



5. Use a graphing calculator to determine the zeros of each function. Sketch each graph using the zeros and *y*-intercept. Then, write the equation of the function in factored form.

a. $h(x) = x^2 - 8x + 12$

zeros: _____

factored form: _____

b. $r(x) = -2x^2 + 6x + 20$

zeros: _____

factored form: _____

c. $w(x) = -x^2 - 4x$

zeros: _____

factored form: _____

d. $c(x) = 3x^2 - 3$

zeros: _____

factored form: _____

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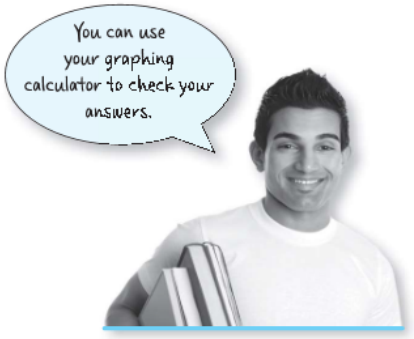
6. Determine the zeros of the function. Write the function in factored form if it is not already in factored form.



a. $f(x) = (x - 2)(x - 7)$

b. $v(x) = x(2x + 6)$

c. $g(x) = (x + 1)(5 - x)$

d. $p(x) = (-9 - 3x)(x + 4)$

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Be prepared to share your solutions and methods.